

Keywords: sociophysics,dynamics, opinion formation, religion, language,

On religion and language evolutions seen through mathematical and agent based models

M. Ausloos^{1,*}

¹*GRAPES@SUPRATECS, ULG, B5a Sart-Tilman, B-4000 Liège, Euroland*

(Dated: today)

Religions and languages are social variables, like age, sex, wealth or political opinions, to be studied like any other organizational parameter. In fact, religiosity is one of the most important sociological aspects of populations. Languages are also obvious characteristics of the human species. Religions, languages appear though also disappear. All religions and languages evolve and survive when they adapt to the society developments. On the other hand, the number of adherents of a given religion, or the number of persons speaking a language is not fixed in time, - nor space.

Several questions can be raised. E.g. from a macroscopic point of view : How many religions/languages exist at a given time? What is their distribution? What is their life time? How do they evolve? From a "microscopic" view point: can one invent agent based models to describe macroscopic aspects? Does it exist simple evolution equations? How complicated must be a model?

These aspects are considered in the present note. Basic evolution equations are outlined and critically, though briefly, discussed. Similarities and differences between religions and languages are summarized. Cases can be illustrated with historical facts and data. It is stressed that characteristic time scales are different. It is emphasized that "external fields" are historically very relevant in the case of religions, rending the study more "interesting" within a mechanistic approach based on parity and symmetry of clusters concepts. Yet the modern description of human societies through networks in reported simulations is still lacking some mandatory ingredients, i.e. the non scalar nature of the nodes, and the non binary aspects of nodes and links, though for the latter this is already often taken into account, including directions.

From an analytical point of view one can consider a population independently of the others. It is intuitively accepted, but also found from the statistical analysis of the frequency distribution that an attachment process is the primary cause of the distribution evolution in the number of adepts: usually the initial religion/language is that of the mother. However later on, changes can occur either due to "heterogeneous agent interaction" processes or due to "external field" constraints, - or both. In so doing one has to consider competition-like processes, in a general environment with different rates of reproduction. More general equations are thus proposed for future work.

PACS numbers:

I. INTRODUCTION

Language and religion like sex, age, wealth, job, political affiliation, ... can be considered to characterize a human individual in its community [1, 2]. Other characteristics may serve to identify a human being, like through its music interests [3, 4] or its previous or present health conditions; see typical questions "who are you?" on visa application forms. These are all examples of some sort of "degrees of freedom" as so often found in statistical physics of critical phenomena [5] and dissipative structures [6], self-organizing or not, in order to describe phase symmetries. Thus statistical physics ideas can find an interesting role outside physics [7, 8] not only in econophysics but also in research pertaining to attitudes, behaviors, opinion formations, etc., as in socio-economic studies. In particular investigations on religion and language distributions, on one hand, and their evolution, on the other hand, can be studied as if such a degree of freedom is a variable defining some thermodynamic state, as a function of time and space [9]. These so called microscopic characteristics, can be studied through auto-correlations, or correlated with any other variable or "parameter" characterizing a population made of so called agents. The results can be next compared with available global data on macroscales.

Several pertinent questions can be raised, e.g. from a "macroscopic" point of view : (i) How many religions/languages exist at a given time? (ii) How are they spatially distributed ? ... From a "microscopic" view

*Electronic address: marcel.ausloos@ulg.ac.be

point: (iii) How many persons consider that they practice one religion/language? (iv) Does the number of adherents increase or not, (v) and how? and maybe why? (vi) Thus what are, if any, the basic equations ? (vii) Last but not least is there some possible modelization, through some agent based model?

However in this limited size contribution it is impossible to present a full state of the art concerning present studies by sociophysicists on languages and/or religions. Only a few topics can be tackled. The selection is admittedly biased and mainly due to a serious lack of competence of the author in front of huge research field activities. In Sec. II the differences and similarities between religions and languages are emphasized, with some stress put on the difference in time scales and the external field constraints. In Sec. III, basic equations are written in to illustrate fundamental approaches to religion and/or language evolutions, without going into details. Many reviews exist elsewhere.

Sec. IV serves to indicate that human networks are more complex than those studied by sociophysicists presently performing simulations of agent based models. In particular the notion of node intensity should appear to be relevant in view of the considerations in the previous sections. In Sec. V a more general set of equations is thereafter proposed and illustrated through an example involving the competitions between a few "degrees of freedom" of different types, like if there are describing struggles between ideologies, in a general sense. A conclusion and suggestions for future work follow in Sec. VI

II. SIMILARITIES AND DIFFERENCES

Through this section, it is hoped to outline a few "differences", also recognizing similarities, between languages and religions, from both an anthropological and a physics point of view. This is summarized through several items in Table I which are briefly commented upon [10]. This should serve as a perspective or input into modeling their sociological features in Sec. III and Sec. V.

For example, the *origin* of languages and/or religions cannot be easily discussed as a simple statistical physics process. Basic anthropological studies, like those of refs.[11–16], or refs. [17–20] are sometimes considered as hypotheses or theories. It is accepted that languages could not have originated as they are without a physiological modification of the human animal. Moreover, diversification is not only due to events like the Babel Tower construction, but is also rather due to different environmental, economic and sociological conditions. Similarly, religions could not originate from myths subsequent to the "scientifically unexplained" observation of natural phenomena. However the "final result", i.e. the appearance of a language or a religion [21] can be imagined by a physicist to correspond to a nucleation/growth process through homogeneous or heterogeneous fluctuations, *exordium* to competition between surface and volume free energy terms. It is relevant to point out to the attempt by Sznajd-Weron to demonstrate how to predict the initial concentration necessary for the fluctuation to grow to a viable size [22] in an opinion model of societies. Nevertheless it will take some time before one can connect such (microscopic) fluctuations (... of what ?) to the truly anthropological approaches. Always containing very profound and elaborate thinking.

First let it be recognized that the definition of a religion or an adherent (or adept) might not be accepted univocally, but the same can be said about languages. It is much debated between scholars on how to define a religion or a language. Like many, one can put on the same footing religions, philosophies, sects and rituals [20]. It is accepted, e.g., that a sect is a legally defined entity, - which in fact depends on the country where the *legal* definition is made. *Idem*, one could distinguish between adherents or adepts; there are also agnostics, atheists or "not concerned". Annoyingly there are various denominations for "closely related" religions which can impair data gathering and subsequent analysis. A similar set of considerations exists when discussing languages and dialects, slangs, etc.: e.g. there are three practical definitions of a language in [23]. The variety of religions and languages is surely enormous, to the point that they may even appear as fully individualistic ones, and only gathered into bins for defining communities.

It is then a habit of human beings to find roots and to connect to some important feature or on the contrary be singled out. Thus finding some genealogical-like trees for languages through their vocabulary or/and grammar, e.g. see [24–27], or similarly for religions, e.g. see [28] can be done through various historical facts or measures. Such work on so called language trees is done like in many approaches about hierarchical systems. Yet it seems forbidden at this level and to-day to rank religions through any other indicators than materialistic ones, i.e. having led to some quantitative measure, - in contrast to written texts in some language, including translations, for which it is more common to discuss their "quality" or "values". One should be careful at another level: take the example pointed out by Nash [29] in the case of christianity. He recalls that in searching for connections with other beliefs, scholars who first use a given terminology to describe e.g. pagan beliefs and practices, then marvel at the awesome parallels they think they have discovered. See also [30]. Nevertheless although it might be an audacious suggestion to consider the hierarchy of languages and religions when describing them through network schemes (see below) , it might be considered to use a measure, based on concepts, as the average overlap index (AOI) for some ranking [31–33], as done for countries in econophysics studies. This has led to some original observation about communities.

It should be stressed that statistical physics needs data in order to produce some thought and answer a few questions.

See the difficulty of obtaining and analyzing data in sociology studies. The data which is often used for describing the language distribution comes from [34, 35]; that for religion has been taken from [36, 37]. However there are many other places where some data can be obtained, including detailed geographical ones [38, 39]. Interestingly one can also connect to data obtained taking into account various economic aspects of religions [40], maybe not so easily about the economics of languages.

Interestingly it is found, by survey takers, that there are about 6000 languages and 3000 religions. One may wonder why the order of magnitude is similar. The more so when it is absolutely clear that one can be a polyglot, but it is *very* difficult to be polyreligious, - even though the caveat pointed out here about holds much strongly here: religion being sometimes *very* individualistic. Therefore the log-normal plots of the *probability distribution function* (PDF) of the number of practitioners of a language [41, 42] or a religion [43] are very strikingly similar, including on the value of the large tail, i.e. respectively -1.42 [44] and - 1.67 [45]. In both cases, the effect of the enhanced number of languages for very small sizes is nevertheless very perturbing, in particular since it announces the forever disappearance of rare cases.

A critical view of this data has to follow: in [43] a break was noticed in the PDF at 10^7 adherents, indicating an overestimation of adepts/adherents in or by) the most prominent religions, or a lack of distinctions between denominations, for these, - as can be easily understood either in terms of propaganda or politics, or because of the difficulty of surveying such cases precisely. Yet one paradoxical surprise stems in the apparent precision of the data. E.g., in several cases, the data in [36] seems to be precise up to the last digit i.e., in mid-2000, there are 1057328093 and 38977 roman catholics and mandeans respectively. In strong contrast there are 7000000 and 1650000 wahhabites and black muslims respectively, numbers which are quite well rounded. Thus a mere reading of the numbers warns about the difficulty of fully trusting the data. Nevertheless the analysis should be pursued bearing this *caveat* in mind. One can here point out an application of PDF studies on Jehovah witness activities, - as if they were economic ones [46].

After having examined the global aspects, let us turn toward more microscopic considerations, i.e. the agents themselves, the role of opinion leaders and the external field effects on agents.

From this time point of view, and stressing the time scale, one can notice that a religion can seem to appear rather instantaneously, often as a so called sect, at the beginning, and its number of adherent can grow steadily (see the recent Mormon or Rastafarianism case) or not. New religions, necessarily called sects at their beginning, appeared after the second world war in Japan. Many disappeared. But in other cases, like the Black muslims, Bahai, Mormons, Sokka-Gakkai, Universal Church of the Kingdom of God (founded in 1977, it has already more than 2 millions adepts), Jehovah witnesses, the Scientology church, etc. New languages can also appear, for very specific reasons: let Esperanto, FORTRAN and other computer "languages", be mentioned here [47, 48]. The notion of nucleation through homogeneous or heterogeneous fluctuations, in some imposed (thus external) field, could be thought of. A religion can also rather quickly disappear [49], like the Antoinists in some coal mine regions of Western Europe, because its social role or its feeding ground disappears. Both cases though quite interesting are actually outside the realm of this paper. Yet the life time, or aging, of a religion can be studied, through the number of adherents, surely for modern times.

Notice the interesting fact when strong fluctuations arise: history is full of examples of individuals or entire groups of people changing their religion, - for various reasons: following the "leader", e.g. Constantinus, Clovis, or not changing at all under "external pressure", leading to martyrdom, or like at inquisition time, or following a fatwah, but also cases of "internal pressure" (Khazars [50], maybe) or so called adaptation under proselytism action, e.g. sun worshipping Incas in presence of catholic missionaries, Zoroastrians in Persia and Bogomils in Bosnia in presence of Muslim Arabs, etc. Such a competition through agent interactions or under "external field conditions" exist in many religion cases indeed. Thus, the number of adherents can evolve drastically due to such various conditions [51, 52], independently of the population growth size. Notice that it can be also a "legal field" which decides upon the use of a language. François I, King of France, decided in Villers-Coteret in 1529 that the true french was the one *spoken* in Isle of France, and nothing else. Back to religion history, a quarter century later, after the Peace of Augsburg treaty (1555), the ruling prince could decide in Germany whether his territory is protestant or catholic: *Cuius regio, eius religio*. In all cases, that does not mean that everybody became a christian in the Roman Empire nor everybody spoke french in what was France, at that time, but it could lead to intolerance, wars of religions, and massacres. One can recall the case of "Matines Brugéaises" or "Brugse Metten" (1302) when flemish "peasants" killed the french nobles, recognized as such, because they could not pronounce correctly "schild and vriend". Other cases are that of the red khmers killing Vietnam educated intellectuals in Cambodia and that of Gileadites killing Ephraimites at a Jordan ford. Yet it is hard to find massacres due to "language heresy", though discrimination is known due to language conflicts, - see the problems of the French speaking community around Brussels, Belgium ! or the contempt of european portugese for the brazilian portugese; kurd and armenian languages in Turkey; not too long ago, french in Canada, etc.

However religious (so called) heresy and differences has led to many more deaths: no need to mention Iran, China,

TABLE I: Comparing languages and religions through a few similarities and differences

	Languages			Religions		
	Refs.					Refs.
origin	[11–13]	physiology	"Babel Tower"	natural phenomena	myths	[2, 17–19]
variety	[14, 23]	huge:	dialects, slangs	huge:	denominations, sects	
semantics	[16, 23]	grammar	vocabulary	images	rituals	[20]
agents	[14, 15]	multilingual	frequent	polyreligious	rare	
impact factor		citations	libraries	"saints"	worship sites	
data set	[34, 35]		WALS		WCT, WCE	[36, 37]
number			more than 6000		more than 3000	
PDF	[68]		\sim log-normal		\sim log-normal	[43]
PDF tail	[41, 44]		- . 1.42		- 1.67	[43, 45]
genealogy	[24, 25, 27]		tree		tree	[28]
hierarchy			(AOI ?)		(AOI !)	
opinion leaders		authors	teachers	(high) priests	witches, shamans	
time scales						
	[61, 62]	nucleation slow growth decay	slow François I	nucleation Contantine, Kazars decay	fast fast through avalanches	[49]
applied fields		Brugse Metten competition enforced civilization	discrimination media ... minorities	inquisition competition proselytism	Bogomils, Cathars economics often, strong decay, revival	[40, 52]
network	[55–57]		small world		IDP-DED	[54]

Palestine, Balkan. This might be due to the fact that religions can serve as ground or excuses for political union: see Poland under the communist regime. Notice that attempts to suppress some "heresy" through harsh means does not always work: see the rise of protestantism in Great Britain even through strong means [53], though inquisition had some effect [51, 52].

See also a mapping of a sometimes called (religious or scientific) controversy on networks made of Intelligent Design Proponents (IDP) and Darwinian Evolution Defenders (DED) in order to analyze the local and global structural properties [54] of such strongly diverging opinion networks. See also the approach toward linguistics within the network concepts, not necessarily involving agents, in [55–57].

Yet, it has been found in [43] that empirical laws can be deduced for the number of adherents, i.e. the *probability distribution function*. Two quite different statistical models were proposed, both reproducing well the data, with the same precision, one being a preferential attachment model [58], like for heterogeneous interacting agents on evolving networks, e.g. it is more likely that one has the religion of one's mother or neighbor..... (leading to a log-normal distribution), another based on a "time of failure" argument (leading to a Weibull distribution function).

III. BASIC EQUATIONS OF EVOLUTION

The most fundamentally relevant variables are thus accepted to be the number of practitioners of a language/religion [59] Only this number is treated here as the physics object/measure. Thus a language/religion is hereby considered as a (socially based) variable to be so studied like any other organizational parameter for defining a thermodynamic-like state. Since a religious state [60]

is more individualistic than a linguistic state, one can better define the religious adherence of an agent than the linguistic one. Indeed one can hardly be multi-religious but one can be a polyglot. Of course one can switch easily, i.e. through "conversion", from one religious denomination to another; this is not so easy in language cases, - except through miracles. Thus the observation time of a religious state needs careful attention in surveys, but is less so drastic for language studies, as already hinted here above.

The dynamics of world's languages, especially their disappearance [61, 62] has been recently considered through a Verhulst time t evolution equation [63] for the number density ρ_i of practitioners of some language i

$$\frac{\partial \rho_i}{\partial t} = r_i \rho_i \left(1 - \frac{\rho_i}{C_i}\right) \quad (1)$$

where C_i is the carrying capacity of the environment for the population speaking the i language and r_i is a positive or negative growth rate.

It was soon discussed that the evolution of one language must take into account some competition with other languages [56, 57, 64–68], but also geographical constraints [69] and social structures [70], sometimes extending the concepts toward hard evolutionist views [71, 72]. Whence the Verhulst equation must be supplemented by a diffusion equation

$$\frac{\partial \rho_i}{\partial t} + \text{div} \vec{j}_i = C_i \quad (2)$$

where \vec{j}_i describes the net flux of the population speaking the language i into some area or in terms of a diffusion coefficient along some x axis

$$\frac{\partial \rho_i}{\partial t} = D_i \frac{\partial^2 \rho_i}{\partial x^2}. \quad (3)$$

There are several review papers like [66–68] discussing such cases.

A set of similar equations for religions can be written when attempting to quantify some religion dynamics [43, 73–76] through the number of adepts. A population growth-death equation has been conjectured indeed to be a plausible modeling of such religion evolution dynamics, in a continuous time framework. The time evolution of several "main" religions was considered to be described, at a so called microscopic level, along the lines of the Avrami-Kolmogorov equation describing solid state formation in a continuous time framework [77]. The solution of which is usually written as

$$f(t) = 1 - e^{-Kt^n} \quad (4)$$

where $f(t)$ is the volume fraction being transformed from one phase to another; K and n are adjustable parameters.

A priori in analogy with crystal growth studies [77, 78], we have considered that a microscopic-like, continuous time differential equation can be written for the evolution of the number of adherents, in terms of the percentage with respect to the world population, of the world main religions, as for competing phase entities in Avrami sense

$$\frac{d}{dt}g(t) = \gamma t^{-h}[1 - g(t)]. \quad (5)$$

It can be solved easily giving the evolution equation for the fraction $g(t)$ of religion adherents in terms of a (positive for growth process) rate (or scaling) parameter to be determined, and a parameter h measuring the attachment-growth (or death) process to be deduced in each case, in a continuous time approximation. It should be emphasized that religions have appeared at some time t_0 which is somewhat unknown, or to say the least, loosely defined, due to a lack of historical facts but also due to the inherent process of the creation of a religion. In both, religion and language, cases, the initial condition (IC) which must be imposed in resolving the above equations is a major difficulty. Fortunately one might have some measure of ρ_i at some given time before integrating.

Thus to address some of these issues, classical scientific steps can be followed as in physics investigations [43, 79, 80]. *Accepting* as valid the number of adherents of religions from various surveys, "empirical" data can be subsequently analyzed. Examples of religions were found for which the number of adherents is increasing (e.g., Islam), decaying (e.g., Ethnoreligions) or rather stable (e.g., Christianity and Buddhism), - all giving interesting values for h . This has led to a more complete and more detailed analysis of the values of h and of its meaning for 58 "time series" [81] in [79, 80]. Anomalous, or unexpected h values have been attributed to so called external field conditions, somewhat based on intuition.

However the main criticism of these equations is that they do not fully take into account social and historical conditions, - in some wide sense, the external/political fields and the environment, in a loose sense. For example in the case of the Antoinist which was founded by Louis Antoine at the end of the XIX-th century in some Liège suburb and had a number of adherents up to about 200 000, after 50 years of existence or so, and nowadays decays, it is found that the evolution equation is better represented through a rate equation which takes into account an external (sociological or not) condition through a parameter, c , i.e.

$$\frac{d\rho}{dt} = b \left(\frac{\rho}{t} \right) - c \rho \quad (6)$$

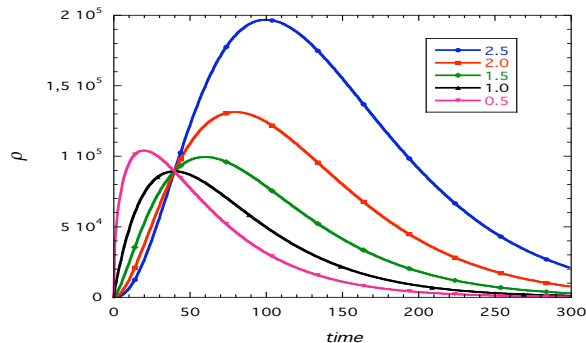


FIG. 1: Graph representing the evolution in the number of adepts under grow/decay conditions, as in Eq.(8), as for the Antoinist. The y - and x - axis values are arbitrary; for the display $t_1 = 40$ and $\alpha = 20$ values were chosen; the value of the b exponent is indicated in the inset

such that the evolution of ρ reads

$$\rho(t) = a \left(\frac{t}{t_1} \right)^b e^{-ct} \quad (7)$$

where a is an integration constant which can be adjusted. This equation states that the relative rate of change of the numbers of adherents $\frac{d\rho}{\rho}$ is proportional to $\frac{dt}{t}$ instead of to $(1 - \frac{\rho}{C}) dt$. and a scaling time t_1 . For a proper display (Fig.1) a change of variables is made such that the previous equation reads

$$\rho(t) = A \left(\frac{t}{t_1} \right)^b e^{-\frac{t}{t_1} + \alpha} \quad (8)$$

Other formulations can be found that would present such a maximum after an appropriate time; they would need to be investigated and interpreted.

In concluding this section it is fair to point out that the spreading of languages and religions can be mapped into some approach like epidemics modeling [52]. At the same time, it can be thought that the spreading is similar to market sharing. Indeed the conclusions of Roach [52] when distinguishing between the behaviors of Cathar heresy in France and that of Bogomils in Bulgaria can be mapped either as an epidemic control or a market share battle depending on whether or not one looks at the overall result or at the cause of the result. Other spreading taking into account asymmetric grow and environmental conditions, can be found in anthropological research, see e.g. Paleindian spreading in [82]. In most of these cases, the grow is analogous to crystal growth [83–88] conditions.

IV. NETWORKS

A. Cluster expansion

In physics one should study the response of the system to intrinsic or extrinsic fields. In a mechanistical approach, the population of agents through a free energy F , Hamiltonian formalism or Langevin equation. In such a way, one would develop the quantity of interest as a series in terms of clusters, e.g., ordered along the increasing size of the cluster according to the number of spins in the cluster $\langle \dots \rangle$, as in

$$\exp \left[-\frac{F - F_0}{kT} \right] = -\sum_i H_i S_i - \sum_{\langle ij \rangle} J_{ij} S_i S_j - \sum_{\langle ijk \rangle} K_{ijk} S_i S_j S_k - \dots \quad , \quad (9)$$

in obvious notations, i.e. each spin S_i representing an adept in an external field H_i and interacting with another adept through some interaction J_{ij} , etc., or similarly

$$\frac{\partial \Phi_i}{\partial t} = A_{ij} \Phi_j + B_{ijk} \Phi_j \Phi_k + C_{ijkl} \Phi_j \Phi_k \Phi_l + \dots \quad , \quad (10)$$

for some information flux Φ_i . A vector generalization is immediately thought of.

B. Node intensity

Although language is primarily *spoken*, most of the relevant research is about its *written* forms of communication, - which have significantly different sets of properties (known as registers) (e.g., [89]) than spoken forms. It occurs that Web documents e.g. come from a wide range of styles, while blogs probably tend to be relatively informal, some being close to spoken language. In contrast, academic web sites contain large collections of very formal documents, such as research papers in e-journals, online copies of computer documentation, and university rules and regulations. Nevertheless, they also contain less formal genres such as personal home pages. Nevertheless one should clearly precise what is meant by "language investigations", since one could wonder whether someone pretending to be a language practitioner, indeed understands, reads, speaks, writes, ... the language. Moreover it seems relevant to question the quality level of the competence.

Similarly the time dependence of the *number* of adherents can be considered to be a very restrictive, and even wrong, way to "measure" the evolution of a religion. One should also "weight" the level of adherence to a religion. For example, one could try as for languages to define a religion through its member intensity of participations in rituals, and in practicing the principles. Indeed, one could distinguish between *adepts* and *adherents*. An adept definition can be found in [90]

One can measure diverse quantities related to the religious effect as well. Moreover, one could consider religion dynamics from another ensemble of points of view or *indicators* [91]. As there are several definitions of a language [23], similarly one could also define what a religion "is" in different ways [20, 92, 93].

Thus it is clear that a religious/language "adherent" instead of being an analog of an up or down spin, is rather a vector for which each element can be a quantity measuring some *value* like one of those considered in sociology, i.e. a "quality". The measure of this quality being controversial depends on whether it is a self-assessment, on whatever scale, or a direct measure of some type. This make modelling very much complex.

No need to say that physicists are not the first ones to reflect on variability in religion distribution or adherence level. One may find already such considerations in books and papers by specialists of the history or sociology of religions, e.g. [20].

Therefore it is emphasized that one should describe society through agents having degrees of freedom which are both qualitative and quantitative in nature. Each node of the network becomes a quite complicated mathematical entity, surely not a scalar. Each node can be better considered as a vector node, each component being like a degree of freedom, represented by a tag, itself having an intensity, like on some energy level in a molecule. Interactions between the various degrees of freedom of different nodes can occur, through a matrix form; each link having a weight and a direction. That interaction matrix might not be symmetric. The more so if time delays are taken into account. Should one say, like molecules made of ions, or nuclei made of elementary particles interacting in complex ways? Should one at this stage recall Haven [94, 95] description of economy and economic agents along quantum mechanical-like concepts? Indeed quantitative and qualitative dynamical evolutions of agents and groups ("denominations") can find some theoretical source in many competition and organization physics models. Next, Potts vector or ferroelectric type (Hamiltonian) models can be imagined for describing an ensemble of religious agent state or evolution.

Thereafter but *exordium* it seems natural to go back to the classical Verhulst ideas and its extensions for prey-predator problems in a Lotka-Volterra model by introducing some realistic conditions on the growth ratios and on the interaction coefficients between the agents in the populations [96, 97]. - Indeed there is an obvious analogy with the problem of extinction of populations [98], with those of religions [43, 79, 80] and languages [62], and also to the very modern question of internet governance [99].

V. GENERALIZED MODEL

Leaving aside the networking description of the society, let us consider a set of Lotka-Volterra equations to describe the "struggles between ideologies", religions or languages, i.e. in a very general sense. It has been proposed to consider the following scheme [100, 101].

Let us consider a set ("area") with a population of N agents. Let the population be divided into $n + 1$ factions or communities: each faction has a specific ideology, such that the number of members in the corresponding community is N_1, N_2, \dots, N_n , but with a singled out fraction N_0 of people which are considered to be *not* concerned, not following an ideology, not speaking any language, at a given time. By definition at each time t ,

$$N = N_0 + \sum_{i=1}^n N_i \quad (11)$$

The overall population $N(t)$ is assumed to evolve according to the generalized Verhulst law

$$\frac{dN}{dt} = r(t, N, N_\nu, p_\mu) N \times \left[1 - \frac{N}{C(t, N, N_\nu, p_\mu)} \right] \quad (12)$$

where N_ν represents the set N_1, \dots, N_n while $p_\mu = (p_1, \dots, p_m)$ is a set of parameters describing the environment, and r is the overall population growth rate; which can be positive or negative.

The growth process is constantly disrupted by small extinction events, as in [102], monitored through $r(t, N, N_\nu, p_\mu)$. As before $C(t, N, N_\nu, p_\mu)$ is the carrying capacity in the area.

A. General remarks

In every community i one has to account for the following processes: deaths, dissatisfaction, unitary conversion, and binary conversion.

1. First, the number of followers of an ideology can decrease through death or dissatisfaction with the ideology, i.e. through a term $r_i N_i$, where $r_i \leq 0$. In general $r_i = r_i(t, N, N_\nu, p_\mu)$.
2. Conversion from one ideology to another can be made *without* direct contact between the followers of different ideologies. This type of conversion happens through the information environment of the population. In order to model the "unitary" conversion we assume that the number of people converted from ideology j to ideology i is proportional to the number N_j of the followers of the ideology j . A field f_{ij} coefficient characterizes the intensity with which this conversion from j to i occurs. Observe that f_{ij} is not necessarily a scalar. The corresponding modeling term is $f_{ij} N_j$; $f_{ii} = 0$. In general

$$f_{ij} = f_{ij}(t, N, N_\nu, p_\mu, C)$$

3. A conversion to the i -th ideology can occur because of direct interaction between a member of this i ideology with a member of some other j ideology. It can be at first assumed that the intensity of the interpersonal contacts is proportional to the numbers N_j and N_i of the followers of the two ideologies. The coefficient characterizing the intensity of the binary conversion is b_{ij} . The larger is b_{ij} , the more people are converted to the i -th ideology. In general the binary conversion coefficients can be $b_{ij} = b_{ij}(t, N, N_\nu, p_\mu, C)$. Notice that if b_{ij} is finite, then $b_{ji} = 0$, since one converts from j to i . Of course $b_{ii} = b_{jj} = 0$: there is no self-conversion.
4. Another conversion has also to be considered. It is possible that someone can convert to the ideology i simply by interacting with two members of ideology j and k . As above one can assume that the relevant term is proportional

to the numbers N_j and N_k of the followers of the two interacting ideologies. The coefficient characterizing the intensity of this (still called) binary conversion is t_{ijk} . The larger is t_{ijk} , the more people are converted to the i -th ideology. In general the relevant binary conversion coefficients can be $t_{ijk} = t_{ijk}(t, N, N_\nu, p_\mu, C)$. Of course $t_{iii} = 0$: there is no self-conversion. Also t_{iik} does not exist, but t_{ikk} and t_{ijj} can be finite and are factors of N_k^2 and N_j^2 respectively. Other coefficients do not exist since one converts toward i .

In general one can have a co-evolution of the environment and the populations with some feedback, i.e. one might have $p_\mu = p_\mu(N, N_\nu, C, t)$, but this will not be discussed here.

In the simplest version of the model namely the case in which all the coefficients are time and p_μ independent, the model system becomes geared by

$$N = N_0 + \sum_{i=1}^n N_i \quad (13)$$

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{C} \right) \quad (14)$$

$$\frac{dN_i}{dt} = r_i N_i + \sum_{\substack{j=0 \\ j \neq i}}^n f_{ij} N_j + \sum_{\substack{j=0 \\ j \neq i}}^n b_{ij} N_i N_j + \sum_{\substack{j=0 \\ j \neq i}}^n \sum_{\substack{k=0 \\ k \neq i}}^n t_{ijk} N_j N_k \quad (15)$$

A few remarks are in order.

Notice that arbitrary values are not allowed for the coefficients of the model. They must have values such that N, N_0, N_1, \dots, N_n be nonnegative at each t .

Terms could apparently be observed to be similar when they refer to the same coupling. Indeed a $b_{ii} N_i^2$ term seem to be of the same nature as $t_{ijj} N_j^2$. One should not be misled, since the former does not exist, and the latter can never read $t_{jii} N_i^2$, since it does not pertain to the evolution equation of N_i . Nevertheless it is emphasized that the N_j^2 term, usually said of higher order, *must* be conserved in order to maintain correct symmetry conditions.

Moreover let us consider the i -th population and the binary conversion characterized by the coefficients t_{ijk} and t_{ikj} where j and k are different from i . One could at first think that $t_{ijk} N_j N_k$ and $t_{ikj} N_k N_j$ describe one and the same process in which the interaction between followers of the j -th and k -th ideology leads to a conversion of these to the i -th ideology. In general however one should not identify the two terms. In so doing in the general model we retain one additional degree of freedom, i.e., that which allows to distinguish between the ideology that is of the initiator of the interaction and the ideology of someone who is apparently simply a participant in the interaction.

Concerning the content of the f_{ij} , b_{ij} , and t_{ijk} parameters, one should be aware that they can arise from two ingredients. On one hand the intended final state i maybe attained through imitation and the (positive or negative) influence of neighbours, - along usual majority/minority games, but on the other hand a *social value* can also be the cause of a change of ideology, i.e. a shift toward a *more rewarding socially* ideology. The exact form of these coefficients should take such considerations into account in further studies.

Finally, these equations should be completed by two terms if one wants to describe real situations. The first one should be a constant (c) term, as introduced in Eq.(8) and a space dependent term of a diffusive type, $D_i \frac{\partial^2 N_i}{\partial x^2}$ as in Eq.(3). The former allows a decay due to say socio-historical circumstances, the latter allows to describe motion of populations in and out the area.

Below we shall consider the dynamics of populations of followers of the ideologies for the cases of presence of 2 ideologies in a given area, without diffusion. It will be shown that even without a constant forcing term some intricate situation nevertheless exists.

B. Competition between two ideologies

In this section the competition for adepts that the presence of a second ideology introduces illustrates the above model. It is shown that an additional ideology leads to a measurable conflictual tension.

Let us consider the case of two ideologies with populations of followers N_0 and N_1 . Let for simplicity all parameters be kept constant. One has

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{C}\right) \quad (16)$$

$$\begin{aligned} \frac{dN_0}{dt} = & r_0 N_0 + f_{01} N_1 + b_{01} N_0 N_1 + \\ & (t_{010} + t_{001}) N_0 N_1 + t_{011} N_1^2 \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{dN_1}{dt} = & r_1 N_1 + f_{10} N_0 + b_{10} N_0 N_1 + \\ & (t_{110} + t_{101}) N_1 N_0 + t_{100} N_0^2 \end{aligned} \quad (18)$$

$$N = N_0 + N_1 \quad (19)$$

The system has a fixed point when only an external field is applied for conversion

$$\check{N}_1 = \frac{C f_{10}}{f_{10} - r_1} \quad (20)$$

When agent-agent interaction exists, one finds two fixed points, but only one satisfies $\check{N}_1 \geq 0$. Its value depends on the initial condition, i.e. whether $N_1(0)/\check{N}_1 \geq$ or ≤ 1 . The number of converted agents is always $N_0 = N - \check{N}_1$. As a numerical example let $C = 1$, $r_1 = r_0 = -0.01$ and $f_{10} = 0.02$. Then $\check{N}_1/\hat{N}_1 = 0.6$, i.e. the number of followers of the ideology 0 decreases by 40%.

Fig. 2 shows a typical result obtained from the numerical investigation of Eqs.(16)-(19). A purely inertial growth and its decline are observed for the population of followers of ideology 1, simultaneously with the evolution of the population of followers of ideology 0. After a maximum of the latter, thus its decline, an inertial growth of the number of followers of ideology 1 takes place again. This can be usefully compared with the behavior discussed by [62] and that of Eq.(8), where in the latter case, no competition is assumed.

Even though the time is measured in Monte-Carlo time units, the coefficient is found to be of the reasonable order of magnitude. Indeed anthropologists [82] reports a measure of the birth rates of nomadic hunters and of agricultural settlers to be in the range 0.003-0.03/year: ethnographic evidence suggests that human population can expand at rates of up to 4% per year when colonizing new habitat (quotation from [82]). Thus in the present case, one has a reproduction rate of 0.005, - which seems to be a good order of magnitude. An equivalent order of magnitude, but with a negative sign can be taken for $r_1 = -0.005$; the value $f_{10} = 0.001$, chosen for the display, is somewhat more arbitrary at this time.

These results can be also compared to a model of telecommunication competition on a network made of nodes (customers) [103], some of them having no mobile phone at first and others belonging to one of several systems. The conclusion in [103] allows some interesting reflection by analogy with the cases of languages and religions. I quote from [103]: "schemes targeting local cliques within the network are more successful at gaining a larger share of the population than those that target users randomly at a global scale (e.g., television commercials, print ads, etc.). This suggests that success in the competition is dependent not only on the number of individuals in the population but also on how they are connected in the network.". It should be added that the network in our above investigation is such that all agents are fully connected with each other, i.e. one considers a fully connected graph. This is equivalent to a mean field approximation study. Notice that the links are weighted through the f_{ij} and b_{ijk} coefficients.

C. Tension index

Of course this causes some tension between the ideologies. A measure of this tension can be found through the index

$$T_{i;k} = 1 - \frac{N_i^{(k)}}{\hat{N}_i}, \quad (21)$$

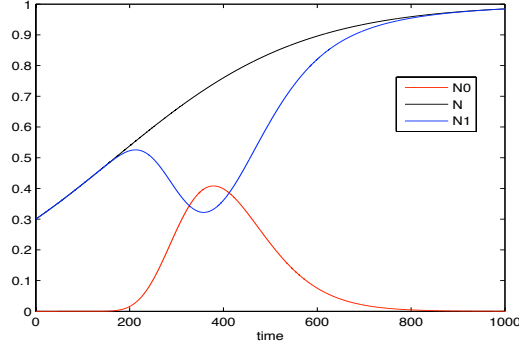


FIG. 2: Illustration of the case of two competing ideologies in populations N_0 and N_1 , in an overall growing (N) population: with $r = 0.005$, $r_1 = -0.005$, and $f_{10} = 0.001$, - thus in the case of no b & no t influenced conversion

where $N_i^{(k)}$ is the population of the followers of the i -th ideology when the k -th ideology is present in the area. If the ideology is alone then $N_1^{(1)} = \hat{N}_1$ and the tension index is $T_{1;1} = 0$. If N_1 decreases because of the competition with the second ideology, then the tension between the ideologies characterised by the tension index $T_{i;k}$ increases. The above definition for the tension holds even if N_1 follows some time dependent trajectory.

Notice that the tension index can be generalized for the case of an arbitrary number of ideologies in the area [100, 101]. Let m ideologies be present.. The tension on the i -th ideology in presence of two other ideologies, k and l can be defined as

$$T_{i;k,l}(t) = 1 - \frac{N_i^{(k,l)}(t)}{\hat{N}_i} \quad (22)$$

where $N_i^{(k,l)}$ is the population of followers of the i -th ideology when the ideologies k and l operate in the area. Similarly, the tension on the i -th ideology in presence of three other ideologies, j , k , l can be measured through

$$T_{i;j,k,l}(t) = 1 - \frac{N_i^{(j,k,l)}(t)}{\hat{N}_i} \quad (23)$$

where $N_i^{(j,k,l)}$ is the number of followers of the i -th ideology in presence of ideologies j , k , l . In such a way one can define a series of indices for the quantification of the tensions among ideologies competing for followers.

VI. DISCUSSION AND CONCLUSIONS

In considering the present studies of physicists on language and/or religion dynamics, one is not only interested in their historical course and anthropological features, but also in attempting to unify the description of natural and social phenomena, for the good of humanity progress.

Here above one has touched upon several aspects, alas neglecting important ones. Mentioned aspects are the size and fraction distribution of denominations in the world. The fluxes are not much studied though. One has insisted on the number of adherents/practitioners, and the role of reproduction rates in the populations, defined though ideologies. The external field influences have been stressed. The conversion through media influence, direct proselytism and indirect, by disgust, conversion have been underlined. Similarities and differences between religion and languages are numerous. For a physicist, the various time scales are of interest. Same for the basic nucleation-growth problem. General differential equations can be written in the spirit of prey-predator systems for describing competitions. The difficulty to introduce quality aspects of language competence and religious practice in simulation through agent based models is an actual challenge. It implies a rethinking of the structure of complex networks.

Many neglected aspects seems to be due to shyness. As the direct comparison of selected word sets of different languages can be used to estimate their historical distance, i.e. the measurement of Levenshtein distances between two languages gives an idea of the time of their first common ancestor, one could define and evaluate religion distances, through various historical indicators. Notice that an interesting phase transition-like problem is the evolution from

polytheism to monotheism. Finally, the economics of religions has already much attracted researchers [40, 46, 49], but it could receive more attention from econophysicists.

In conclusion, mathematical studies and agent based models on religion and languages are fascinating challenges for physicists in the future. Combining research in different fields, like anthropology, economy, sociology and so called hard science will remove barriers between fields and thus will allow not only scientific progress but also provide bases for human better understanding of its life in communities.

Acknowledgments

Thanks to P. Clippe, F. Petroni, S. Pirotte, G. Rotundo, A. Scharnhorst, D. Stauffer, J. Steele, N. Vitanov for various critical comments and sometimes joint work on this subject. Moreover this paper would not have its form nor content without the persuading influence of E. Haven. Thanks also to the CHEAD organizers for some financial support having allowed some presentation of these ideas in Saskatoon in Aug. 2009.

-
- [1] N. Chomsky, *Reflections on Language* (Pantheon Books, New York, 1975); N. Chomsky, *Aspects of the theory of syntax*, (Cambridge, MA: MIT Press, 1965); N. Chomsky, *Language and Mind*. San Diego, Harcourt Brace Jovanovich (1972).
 - [2] E. Durkheim, *Les Formes élémentaires de la vie religieuse* ; transl: *The Elementary Forms of Religious Life* (1912).
 - [3] R. Lambiotte and M. Ausloos, Uncovering collective listening habits and music genres in bipartite networks, *Phys. Rev. E* **72** (2005) 066107 (11 pages)
 - [4] R. Lambiotte and M. Ausloos, On the genre-fication of Music: a percolation approach, *Eur. Phys. J. B* **50** (2006) 183-188.
 - [5] H.E. Stanley, *Phase transitions and critical phenomena* (Oxford Univ. Press, Oxford, 1971)
 - [6] P. Glansdorff and I. Prigogine, *Structure, Stabilité et Fluctuations*, (Masson, Paris, 1971).
 - [7] D. Stauffer, Introduction to statistical physics outside physics, *Physica A* **336** (2004) 1-5.
 - [8] A. Carbone, G. Kaniadakis, and A. M. Scarfone, Tails and Ties Topical Issue on Physics in Society, *Eur. Phys. J. B* **57** (2007) 121-125
 - [9] J. Mimkes, A Thermodynamic Formulation of Social Science in *Econophysics and Sociophysics*, B. K. Chakrabarti, A. Chakraborti, and A. Chatterjee, Eds. (Wiley-VCH, Berlin, 2006). pp. 279-310.
 - [10] The order of items in the Table is somewhat arbitrary but is intended to go from a general, anthropological-like point of view, to a more statistical physics one. Moreover the list of references is very biased, and does not recognize the enormous amount of works on the subject; surely many of which the author is unaware of, - what he deeply regrets.
 - [11] M. Ruhlen, *A guide to the world's languages*, Stanford University Press, Stanford (1987).
 - [12] S. Johansson, *Origins of Language - Constraints on Hypotheses*, see [http : //www.arthist.lu.se/kultsem/pro/SverkerJohanssonsem.pdf](http://www.arthist.lu.se/kultsem/pro/SverkerJohanssonsem.pdf).
 - [13] J. Diamond, *The Third Chimpanzee: The Evolution and Future of the Human Animal* New York: Harper Perennial, (1992, 2006) 141-167.
 - [14] L. Campbell, *Historical Linguistics: An Introduction*, MIT Press, Cambridge, 1999.
 - [15] J. Nichols, Diversity and stability in languages, in: B. D. Joseph, R. D. Janda (Eds.), *The Handbook of Historical Linguistics*, Blackwell Publishing, Malden/Oxford/Melbourne/Berlin, 2003, pp. 283-310.
 - [16] J. H. Greenberg, Some universals of grammar with particular reference to the order of meaningful elements, in: J. H. Greenberg (Ed.), *Universals of Language*, MIT Press, London, 1963, pp. 73-113.
 - [17] M. Eliade, *A History of Religious Ideas, vol. I, From the Stone Age to the Eleusinian Mysteries*, Chicago, IL: University of Chicago Press (1978).
 - [18] G. Swanson, *The Birth of The Gods: The Origin of Primitive Beliefs* (University of Michigan Press, Ann Arbor:,1960)
 - [19] D. Dubuisson, *The Western Construction of Religion: Myths, Knowledge, and Ideology* (2003)
 - [20] D. C. Dennett, *Breaking the Spell: Religion as a Natural Phenomenon*, Penguin Group, (2006).
 - [21] M. Afzal Upal, Simulating the Emergence of New Religious Movements, *Journal of Artificial Societies and Social Simulation* **8** (2005) 1; [http : //jasss.soc.surrey.ac.uk/8/1/6.html](http://jasss.soc.surrey.ac.uk/8/1/6.html)
 - [22] K. Sznajd-Weron, Instabilities in population dynamics, *Eur. J. Phys. B* **16** (2000) 183-187.
 - [23] J.M. Klinkenberg, *Des langues romanes* (Duculot, Louvain-la-Neuve, 1994)
 - [24] M. Serva and F. Petroni, Indo-European languages tree by Levenshtein distance, *Europhys. Lett.* **81** (2008) 68005
 - [25] F. Petroni and M. Serva, Language distance and tree reconstruction, *J. Stat. Mech.* P08012 (2008).
 - [26] D. Zanette, Self-similarity in the taxonomic classification of human languages, *Advances in Complex Systems* **4** (2001) 281-286.
 - [27] S. Wichmann, P. M. C. de Oliveira, V. Velupillai, A. Muller, D. Bakker, and A. P. Grant, A universal law of language taxonomies, preprint (2008)
 - [28] There is to my knowledge at this time no "minimum spanning tree" presented for religions as there are for languages. However an interesting and artistic one can be found concerning a genealogical aspect of the cistercian monasteries; see a reproduction of the original drawing in Constance H. Berman, *The Cistercian Evolution*. The Invention of a Religious

- Order in Twelfth-Century Europe (University of Pennsylvania Press, Philadelphia, 2000) figure 25, page 105, "Engraving of an eighteenth-century genealogical tree for monasteries for men of the Cistercian Order. (Dijon, Bibl. Mun. 90-098). A religious martyr tree can also be seen in [36].
- [29] R. H. Nash, *The Gospel and the Greeks: Did the New Testament borrow from Pagan Thought?* (Dallas: Word Publishing, 1992), p.126.
 - [30] Yeon-Mu Choi and Hyun-Joo Kim, *Physica A* **382** (2007) 665671.
 - [31] M. Gligor and M. Ausloos, Cluster structure of EU-15 countries derived from the correlation matrix analysis of macroeconomic index fluctuations, *Eur. Phys. J B* **57** (2007) 139-146
 - [32] M. Gligor and M. Ausloos, Convergence and cluster structures in EU area according to fluctuations in macroeconomic indices, *Journal of Economic Integration* **23** (2008) 297-330
 - [33] M. Gligor and M. Ausloos, Clusters in weighted macroeconomic networks : the EU case. Introducing the overlapping index of GDP/capita fluctuation correlations, *Eur. Phys. J. B* **63** (2008) 533-539
 - [34] M. Haspelmath, M. Dryer, D. Gil, and B. Comrie (Eds.), *The World Atlas of Language Structures* (Oxford University Press, Oxford, 2005).
 - [35] B.F. Grimes, *Ethnologue: languages of the world* (14th edn. 2000). Dallas, TX: Summer Institute of Linguistics; www.sil.org.
 - [36] D. Barrett and T. Johnson, *World Christian Trends AD 30 - AD 2200. Interpreting the annual Christian megacensus*. William Carey Library (2001).
 - [37] D. Barrett, G. Kurian, and T. Johnson, *World Christian Encyclopedia (2nd edition)*. New York: Oxford University Press (2001).
 - [38] <http://www.census.gov/prod/www/religion.htm>;
<http://www.census.gov/compendia/statab/>
 - [39] <http://www.census.gov/compendia/statab/cats/population.html>.
 - [40] L. Iannaccone, Introduction to Economics of Religion, *J. Econ. Lit.* **36** (1998) 1465-1496.
 - [41] S. Wichmann, On the power law distribution of language family sizes, *J. Linguist.* **41** (2005) 117-131.
 - [42] P. M. C. de Oliveira, D. Stauffer, F. S.W. Lima, A. O. Sousa, C. Schulze, and S. Moss de Oliveira, *Physica A* **376** (2007) 609-616.
 - [43] M. Ausloos and F. Petroni, Statistical dynamics of religions and adherents, *Europhys. Lett.* **77** (2007) 38002 (4pp)
 - [44] P. M. C. de Oliveira, D. Stauffer, S. Wichmann, and S. M. de Oliveira, A computer simulation of language families, *J. Linguist.* **44** (2008) 659-675 .
 - [45] D.H. Zanette and S.C. Manrubia, Vertical transmission of culture and the distribution of family names, *Physica A* **295** (2001) 1-8.
 - [46] S. Picoli Jr. and R.S. Mendes, Universal features in the growth dynamics of religious activities, *Phys. Rev. E* **77** (2008) 036105.
 - [47] M. Boulton, *Zamenhof, Creator of Esperanto* (Routledge, Kegan & Paul, London, 1960).
 - [48] M. Zemankova and C.M. Eastman, Comparative lexical analysis of FORTRAN code, code comments and English text, ACM Southeast Regional Conference Proceedings of the 18th annual Southeast regional conference, Tallahassee, Florida, 193 - 197 (1980).
 - [49] M. Ausloos, unpublished
 - [50] C. Zuckerman, On the date of the Khazars' conversion to judaism and the chronology of the kings of the Rus Oleg and Igor, *Revue des études byzantines* **53** (1995) 237-270.
 - [51] P. Ormerod and A.P. Roach, The medieval inquisition: scale-free networks and the suppression of heresy, *Physica A* **339** (2004) 645-652.
 - [52] A.P. Roach, unpublished
 - [53] P. Ormerod and A. P. Roach, Social Networks and Ideological Movements in History: Burning and the Rise of English Protestantism, ... preprint.
 - [54] A. García Cantú and M. Ausloos, Organizational and dynamical aspects of a small network with two distinct communities: Neo creationists vs. Evolution Defenders, *Scientometrics* **80** (2009) 457-472.
 - [55] R. Ferrer i Cancho and R.V. Solé, The small-world of human language, *Proc. R. Soc. B* **268** (2001) 2261-2265.
 - [56] S.N. Dorogovtsev and J.F.F. Mendes, Language as an evolving Word Web, *Proc. R. Soc. London B* **268** (2001) 2603-2606;
 - [57] Jinyun Ke, Tao Gong, and W. S-Y Wang, Language Change and Social Networks, Social networks play an important role in determining the dynamics and outcome of language change. ; J. Ke, T. Gong, W. S.-Y. Wang, talk at 5th EVOLANG conference, Leipzig, Germany, March 31, 2004; *Int. J. Mod. Phys. C* , in press.
 - [58] A.L. Barabási and R. Albert, Emergence of scaling in random networks, *Science* **286** (1999) 509-512.
 - [59] It is sometimes hard to know or to be sure whether an adherent, a disciple, ... is truly a member of a religious denomination or church. However this caveat pertains to usual problems encountered in sociological investigations.
 - [60] ... admitting indifference, atheism, agnosticism, ... as a sort of religion, from our point of view.
 - [61] M. Nowak and D. Krakauer, The evolution of language, *Proc. Natl. Acad. Sci. USA* **96** (1999) 8028-8033.
 - [62] D.M. Abrams and S.H. Strogatz, Modelling the dynamics of language death, *Nature* **424** (2003) 900.
 - [63] P.F. Verhulst. *Recherches mathématiques sur la loi d'accroissement de la population*. Nouveaux Mémoires de l'Académie Royale des Sciences et Belles-Lettres de Bruxelles, **18**, 145 (1845).
 - [64] V.M. de Oliveira, M.A.F. Gomes, and I.R. Tsang, Theoretical model for the evolution of the linguistic diversity, *Physica A* **361** (2006) 361-370.
 - [65] M. Thelwall and L. Price: , Language Evolution and the Spread of Ideas on the Web: A Procedure for Identifying Emergent

- Hybrid Word Family Members, *Journal of the American Society for Information Science and Technology (JASIST)*, **57** (2006) 1326-1337
- [66] C. Schulze and D. Stauffer, Microscopic and macroscopic simulation of competition between languages, *Phys. Life Rev.* **2** (2005) 89-116
- [67] C. Schulze and D. Stauffer, Computer simulation of language competition by physicists, in *Econophysics and Sociophysics: Trends and Perspectives*, B.K. Chakrabarti, A. Chakraborti and A. Chatterjee, Eds. (Wiley-VCH, Weinheim, 2006) pp. 311-318
- [68] C. Schulze, D. Stauffer, and S. Wichmann, Birth, survival and death of languages by Monte Carlo simulation, *Comm. Comput. Phys.*, **3** (2008) 271-294
- [69] C. Schulze and D. Stauffer, Competition of languages in presence of a barrier, preprint (2007).
- [70] J. W. Minett, and W. S. Y. Wang, Modelling endangered languages: The effects of bilingualism and social structure, *Lingua* xxxx (2007) ... -...; W. S. Y. Wang and J. W. Minett, Vertical and horizontal transmission in language evolution, *Trans. Philological Soc.* **103** (2005) 121-, [http : //www.isrl.uiuc.edu/amag/langev/](http://www.isrl.uiuc.edu/amag/langev/) ; W. S. Y. Wang, and J. W. Minett, The invasion of language: emergence, change and death, *Trends in Ecology and Evolution* **20** (2005) 263-296.
- [71] F. Hashemi, An evolutionary model of the size distribution of sect/congregations, *J. Evol. Econ.* **10** (2000) 507-521.
- [72] C. Buckley and J. Steele, Evolutionary ecology of spoken language: co-evolutionary hypotheses are testable, *World Archaeology* **34** (2002) 26-46.
- [73] T.M. Johnson and D. Barrett, Quantifying alternate futures of religion and religions, *Futures* **36** (2004) 947-960.
- [74] J. Hayward, Mathematical Modeling of Church Growth, *J. Math. Sociol.* **23** (1999) 255-292.
- [75] J. Hayward, A General Model of Church Growth and Decline, *J. Math. Sociol.* **29** (2005) 177-207.
- [76] O. Shy, Dynamic models of religious conformity and conversion: Theory and calibrations, *Eur. Econ. Rev.* **51** (2007) 1127-1153.
- [77] R. Cloots, N. Vandewalle, and M. Ausloos, Simulations of the kinetic growth of $YBa_2Cu_3O_{7-d}$ grains, *J. Cryst. Growth* **166** (1996) 816-819.
- [78] A. Gadomski, Stretched exponential kinetics of the pressure induced hydration of model lipid membranes. A possible scenario *J. Phys. II France* **6** (1996) 1537-1546.
- [79] M. Ausloos and F. Petroni, On World Religion Adherence Distribution Evolution, [http : //arXiv.org/e – print/](http://arXiv.org/e-print/) : 0801.1010
- [80] M. Ausloos and F. Petroni, Statistical Dynamics of Religions, *Physica A* (2009).
- [81] More precisely there are 56 data sets for specific religions, in the World Christian Encyclopedia (WCE) and World Christian Trends (WCT) reference books [36, 37], most of them being in the *main denomination* bracket, i.e. in the upper part of the PDF as obtained from the surveys taken between 1900 and 2000. Some data refers to 3000 religions which are put together, as "Other Religions" in the WCT tables. Moreover some data refers to *Atheists* and *Nonreligious* persons, [36]. Thereafter for conciseness, those three sets are called "religions".
- [82] J. Steele, J. Adams, and T. Sluckin, Modelling Paleoindian dispersals, *World Archaeology* **30** (1998) 286–305
- [83] [http : //physicsweb.org/articles/news/11/1/1/1](http://physicsweb.org/articles/news/11/1/1/1)
- [84] J. Candia and E. V. Albano, *Int. J. Mod. Phys. C* **19** (2008) 1617-...
- [85] N. Vandewalle and M. Ausloos, Inner Patterns and Front Propagation of a Dynamic Random Impurity Model, *Phys. Rev. Lett.* **77** (1996) 510-513.
- [86] M. Ausloos, N. Vandewalle, and R. Cloots, Magnetic Kinetic Growth Models, *J. Magnet. Magnet. Mater.* **140-144** (1995) 2185-2186.
- [87] N. Vandewalle and M. Ausloos, Lack of universality in two-dimensional multicomponent spreading phenomena, *Phys. Rev. E* **52** (1995) 3447-3454.
- [88] M. Ausloos and N. Vandewalle, Growth models with internal competition, *Acta Phys. Pol. B* **27** (1996) 737-746.
- [89] D. Biber, Variation among University spoken and written registers: A new multi-dimensional analysis. In P. Leistyna & C. F. Meyer (Eds.), *Corpus Analysis: Language Structure and Language Use* Amsterdam: Rodopi (2003) (pp. 47-70). [http : //en.wikipedia.org/wiki/Adept](http://en.wikipedia.org/wiki/Adept)
- [90] C. Herteliu, Statistical Indicators System regarding Religious Phenomena, *J. Study of Religions and Ideologies* **16** (2007) 111-127.
- [92] [http : //www.religioustolerance.org/rel_defn.htm](http://www.religioustolerance.org/rel_defn.htm)
- [93] [http : //www.religioustolerance.org/var_rel.htm](http://www.religioustolerance.org/var_rel.htm)
- [94] A. Khrennikov and E. Haven, Quantum mechanics and violations of the sure-thing principle: the use of probability interference and other concepts, *J. Math. Psychol.*, in press (2009)
- [95] E. Haven, Private information and the 'information function': a survey of possible uses. *Theory and Decision* **64** (2008) 193-228.
- [96] Z. I. Dimitrova and N. K. Vitanov, Influence of adaptation on the nonlinear dynamics of a system of competing populations, *Phys. Lett. A* **272**, 368-380 (2000).
- [97] N. K. Vitanov, Z. I. Dimitrova, and H. Kantz. On the trap of extinction and its elimination, *Phys. Lett. A* **349**, 350-355 (2006).
- [98] A. Pekalski and M. Ausloos, Risk of population extinction from periodic and abrupt changes of environment, *Physica A* **387** (2008) 2526-2534.
- [99] L. N. Olivera, A. N. Proto, and M. Ausloos, Modeling the information society as a complex system, (to be published) (2009).

- [100] N. K. Vitanov, Z. I. Dimitrova, and M. Ausloos, A model of ideological struggle, arxiv....
- [101] N. K. Vitanov, Z. I. Dimitrova, and M. Ausloos, On the dynamics of populations which members can change their properties. arxiv...
- [102] C. Wilke and T. Martinetz, Simple model of evolution with variable system size, *Phys. Rev. E* **56** (1997) 7128-7131.
- [103] E. F. Legara, A. Longjas, and R. Batac, Competition in a social structure, *Int. J. Mod. Phys. C* **20**, (2009) 1-7.